(13)

$$(\partial_t - D\nabla^2)\, g(\bar r,t) = \delta(\bar r)\,\delta(t)$$

$$g(\bar r,t) = \frac{1}{2\pi}\frac{1}{(2\pi)}\int d^2k \int d\omega\, e^{i\bar k\bar r - i\omega t} \qquad g(k,\omega) = \frac{1}{(2\pi)^2}\int d^2k\, e^{i\bar k \bar r}$$

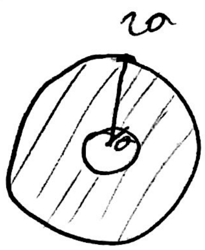
2D

$$\frac{1}{2\pi}\int d\omega \frac{e^{-i\omega t}}{Dk^2 - i\omega} = \frac{\theta(t)}{2\pi}\int d^2k\, e^{i\bar k\bar r}\, e^{-Dtk^2} =$$

$$= \frac{1}{(4\pi D t)^{1/2}}\, e^{-r^2/4Dt} \qquad \begin{array}{l}(k^2 = k_x^2 + k_y^2)\\ (\bar k \bar r = k_x \cdot x + k_y \cdot y\end{array}$$

3D $$g(\bar r, t) = \frac{\theta(t)}{(4\pi D t)^{3/2}}\, e^{-r^2/4Dt} \qquad \langle r^2\rangle \sim t$$

(11)



$$(\partial_t^2 - c^2\nabla^2)\psi(\bar r,t) = 0$$

$$\psi(\bar r,t) = \phi(\bar r)\, e^{\pm i\omega t}$$

$$-\nabla^2\phi_{nm}(\bar r) = k^2_{nm}\phi(\bar r) \qquad \phi_{nm}(\bar r) = R_{nm}(r)\, e^{im\varphi}$$

$$R(r) = A_{nm} J_m(k_{nm} r) + B_{nm} N_m(k_{nm} r)$$

$$A_{nm} J_m(k_{nm} r_1) + B_{nm} N_m(k_{nm} r_1) = 0$$

$$A_{nm} J_m(k_{nm} r_2) + B_{nm} N_m(k_{nm} r_2) = 0$$

buscamos $\det = 0$

$$J_m(k_{nm} r_1) N_m(k_{nm} r_2) = J_m(k_{nm} r_2) N_m(k_{nm} r_1)$$

<u>$m=0$</u> $\quad r_1 = a \quad r_2 = 2a \quad \omega_{nm} = c k_{nm}$ ↑ <u>por ordenador</u>

$a k_{n=1,m=0} = 3'12$; $n=2 \Rightarrow 6'27$; $a k_{n=3} = 9'418$ . . $n=4$ $12'56$

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(13)

$$(\partial_t - D\nabla^2)\, g(\bar{r},t) = \delta(\bar{r})\,\epsilon(t)$$

$$g(\bar{r},t) = \frac{1}{2\pi}\frac{1}{(2\pi)}\int d^2k \int d\omega\, e^{i\bar{k}\bar{r} - i\omega t} \qquad g(\bar{k},\omega) = \frac{1}{(2\pi)^2}\int d^2k\, e^{i\bar{k}\bar{r}}$$

2D

$$\frac{1}{2\pi}\int d\omega \frac{e^{-i\omega t}}{Dk^2 - i\omega} = \frac{\theta(t)}{2\pi}\int d^2k\, e^{i\bar{k}\bar{r}}\, e^{-Dtk^2} =$$

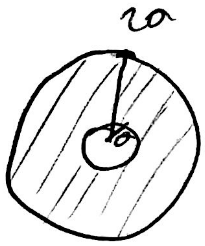
$$= \frac{1}{(4\pi D t)^{1/2}}\, e^{-r^2/4Dt}$$

$(k^2 = k_x^2 + k_y^2)$

$(\bar{k}\bar{r} = k_x \cdot x + k_y \cdot y$

3D $$g(\bar{r},t) = \frac{\theta(t)}{(4\pi D t)^{3/2}}\, e^{-r^2/4Dt} \qquad \langle r^2 \rangle \sim t$$

(11)



$$(\partial_t^2 - c^2\nabla^2)\,\psi(\bar{r},t) = 0$$

$$\psi(\bar{r},t) = \phi(\bar{r})\, e^{\pm i\omega t}$$

$$-\nabla^2 \phi_{nm}(\bar{r}) = k_{nm}^2\, \phi(\bar{r}) \qquad \phi_{nm}(\bar{r}) = R_{nm}(r)\, e^{im\varphi}$$

$$R(r) = A_{nm} J_m(k_{nm} r) + B_{nm} N_m(k_{nm} r)$$

$$A_{nm} J_m(k_{nm} r_1) + B_{nm} N_m(k_{nm} r_1) = 0$$

$$A_{nm} J_m(k_{nm} r_2) + B_{nm} N_m(k_{nm} r_2) = 0$$

buscamos $\det = 0$

$$J_m(k_{nm} r_1)\, N_m(k_{nm} r_2) = J_m(k_{nm} r_2)\, N_m(k_{nm} r_1)$$

$\underline{m=0}$ $\quad r_1 = a \quad r_2 = 2a \quad \omega_{nm} = c k_{nm}$ $\quad$ ↑ por ordenador

$a k_{n=1,m=0} = 3'12$; $n=2 \Rightarrow 6'27$; $a k_{n=3} = 9'418$ .. $n=4$ $12'56$

22

$$(\partial_t^2 - c^2 \nabla^2)\, g(\bar r, t) = \delta(\bar r)\,\delta(t)$$

$$g(\bar r, t) = \frac{1}{(2\pi)^{3/2}} \int d^2k \int d\omega\, \tilde g(\bar k, \omega)\, e^{i(\bar k \bar r - \omega t)}$$

$$g(\hat k, \omega) = \frac{1}{(2\pi)^{3/2}} \int d^3 r \int dt\, g(\bar r, t)\, e^{-i(\bar k \bar r - \omega t)}$$

2

$$\tilde g(\hat k, \omega) = \frac{1}{(2\pi)^{3/2}} \frac{1}{c^2k^2 - \omega^2}$$

$$g(\bar r, t) = \frac{1}{4\pi^2} \int d^2k\, e^{i\bar k \bar r} \int \frac{d\omega}{2\pi} \frac{e^{-i\omega t}}{c^2k^2 - \omega^2}$$

$$f(t) = \frac{1}{2\pi} \int d\omega \frac{e^{i\omega t}}{c^2k^2 - \omega^2} = \begin{cases} 0 & t < 0 \\ \dfrac{\operatorname{sen}(ckt)}{ck} & t > 0 \end{cases}$$

$$g(\bar r, t) = \frac{1}{4\pi^2} \int d^2k\, e^{i\bar k \bar r} f(k,t) = \frac{1}{2\pi} \int_0^\infty k\,dk\, f(k,t) \underbrace{\int \frac{d\varphi}{2\pi} e^{ikr\cos\varphi}}_{J_0(kr)} =$$

$$= \frac{1}{2\pi c} \int_0^\infty dk\, J_0(kr)\, \operatorname{Sen}(kct) = \begin{cases} 0 & ct < r \quad r = |\bar r| \geq 0 \\ \dfrac{1}{2\pi c} \dfrac{1}{\sqrt{c^2t^2 - r^2}} & ; ct > r \end{cases}$$

$$G(\bar r, t, \bar r_0, t_0) \qquad r \to |\bar r - \bar r_0| \qquad t \to t - t_0$$

22

$$(\partial_t^2 - c^2 \nabla^2) g(\bar{r},t) = \delta(\bar{r})\,\delta(t)$$

$$g(\bar{r},t) = \frac{1}{(2\pi)^{3/2}} \int d^2k \int d\omega\, \tilde{g}(\bar{k},\omega)\, e^{i(\bar{k}\bar{r}-\omega t)}$$

$$g(\hat{k},\omega) = \frac{1}{(2\pi)^{3/2}} \int d^3r \int dt\, g(\bar{r},t)\, e^{-i(\bar{k}\bar{r}-\omega t)}$$

2

$$\tilde{g}(\hat{k},\omega) = \frac{1}{(2\pi)^{3/2}} \frac{1}{c^2k^2-\omega^2}$$

$$g(\bar{r},t) = \frac{1}{4\pi^2} \int d^2k\, e^{i\bar{k}\bar{r}} \int \frac{d\omega}{2\pi} \frac{e^{-i\omega t}}{c^2k^2-\omega^2}$$

$$g(t) = \frac{1}{2\pi} \int d\omega \frac{e^{i\omega t}}{c^2k^2-\omega^2} = \begin{cases} 0 & t<0 \\ \dfrac{\mathrm{sen}(ckt)}{ck} & t>0 \end{cases}$$

$$g(\bar{r},t) = \frac{1}{4\pi^2} \int d^2k\, e^{i\bar{k}\bar{r}}\, g(k,t) = \frac{1}{2\pi} \int_0^\infty k\,dk\, g(k,t) \underbrace{\int \frac{d\varphi}{2\pi} e^{ikr\cos\varphi}}_{J_0(kr)} =$$

$$= \frac{1}{2\pi c} \int_0^\infty dk\, J_0(kr)\,\mathrm{Sen}(kct) = \begin{cases} 0 & ct<r \quad r=|\bar{r}|\geq 0 \\ \dfrac{1}{2\pi c} \dfrac{1}{\sqrt{c^2t^2-r^2}} & ; ct>r \end{cases}$$

$G(\bar{r}, t, \bar{r}_0, t_0)$ $\quad r \to |\bar{r}-\bar{r}_0|$

$t \to t - t_0$

(21) H4

$$\left(\left(\frac{\omega}{c}\right)^2+\nabla^2\right) G(\bar r, \bar r_0) = -\delta(\bar r-\bar r_0)$$

$$g(\bar r) = G(\bar r, \bar r_0 = 0)$$

$$\left(\left(\frac{\omega}{c}\right)^2+\nabla^2\right) g(\bar r) = -\delta(\bar r)$$

- $g(\bar r) = \frac{1}{2\pi}\int d^2k\, e^{i\bar k\bar r}\tilde g(k)$; $\tilde g(k) = \frac{1}{2\pi}\int d^2r\, e^{-i\bar k\bar r} g(r)$

$$\tilde g(\bar k) = \frac{1}{(2\pi)^2}\frac{1}{k^2-\tilde\omega^2} \qquad \tilde\omega = \omega/c$$

$$g(r) = \frac{1}{(2\pi)^2}\int d^2k\,\frac{e^{i\bar k\bar r}}{k^2-\tilde\omega^2} = \frac{1}{(2\pi)^2}\int_0^\infty k\,dk\int_0^{2\pi} d\varphi\,\frac{e^{ikr\cos\varphi}}{k^2-\omega^2} =$$

$$= \int_0^\infty k\,dk\,\frac{J_0(kr)}{k^2-\omega^2}\frac{1}{2\pi} \qquad \frac{J_0(kr)}{k^2-\omega^2}$$

$\omega \to \omega^+ = \omega + i0^+$ 10.27.46

$$g(r) = \frac{K_0(-i\tilde\omega^+ r)}{2\pi} = \begin{cases} \frac{+\pi i}{2} H_0^{(1)}(|\tilde\omega| r) & \mathrm{sgn}(\tilde\omega)>0 \\ \frac{-\pi i}{2} H_0^{(2)}(|\tilde\omega| r) & \mathrm{sgn}(\tilde\omega)<0 \end{cases}$$

DLMF 10.27.9 DLMF 10.27.8

$$\boxed{g(\bar r) = \begin{cases} i/4\, H_0^{(1)}(|\tilde\omega| r) & \mathrm{sgn}(\tilde\omega)>0 \\ -i/4\, H_0^{(2)}(|\tilde\omega| r) & \mathrm{sgn}(\tilde\omega)<0 \end{cases}}$$

$$H_0^{(1,2)} \sim J_0 \pm iN_0$$

$$g(r\to 0) = -\frac{1}{4}N_0(|\tilde\omega| r) \to -\frac{1}{2\pi}\ln(r)$$

$$g(r)e^{-i\omega t} \xrightarrow[r\to\infty]{} \begin{cases} \frac{1}{\sqrt r} e^{i\frac{|\omega|}{c}(r-ct)} e^{-i\pi/4} & \omega>0 \\ \frac{1}{\sqrt r} e^{-i\frac{|\omega|}{c}(r-ct)} e^{+i\pi/4} & \omega<0 \end{cases}$$

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21 H4

$$\left(\left(\frac{\omega}{c}\right)^2 + \nabla^2\right) G(\bar{r}, \bar{r}_0) = -\delta(\bar{r} - \bar{r}_0)$$

$$g(\bar{r}) = G(\bar{r}, \bar{r}_0 = 0)$$

$$\left(\left(\frac{\omega}{c}\right)^2 + \nabla^2\right) g(\bar{r}) = -\delta(\bar{r})$$

• $g(\bar{r}) = \frac{1}{2\pi}\int d^2k\, e^{i\bar{k}\bar{r}}\, \tilde{g}(k)$; $\tilde{g}(k) = \frac{1}{2\pi}\int d^2r\, e^{-i\bar{k}\bar{r}}\, g(r)$

$$\tilde{g}(\bar{k}) = \frac{1}{(2\pi)^2}\frac{1}{k^2 - \tilde{\omega}^2} \qquad \tilde{\omega} = \omega/c$$

$$g(r) = \frac{1}{(2\pi)^2}\int d^2k\, \frac{e^{i\bar{k}\bar{r}}}{k^2 - \tilde{\omega}^2} = \underbrace{\frac{1}{(2\pi)^2}}\int_0^\infty k\,dk \underbrace{\int_0^{2\pi} d\varphi\, \frac{e^{ikr\cos\varphi}}{k^2 - \omega^2}}_{\left(\frac{J_0(kr)}{k^2-\omega^2}\right.} =$$

$$= \int_0^\infty k\,dk\, \frac{J_0(kr)}{k^2 - \omega^2}\frac{1}{2\pi}$$

$\omega \to \omega^+ = \omega + i0^+$ 10.27.46

$$g(r) = \frac{K_0(-i\tilde{\omega}^+ r)}{2\pi} = \begin{cases} \frac{+\pi i}{2} H_0^{(1)}(|\tilde{\omega}| r) & \mathrm{sgn}(\tilde{\omega}) > 0 \\ \frac{-\pi i}{2} H_0^{(2)}(|\tilde{\omega}| r) & \mathrm{sgn}(\tilde{\omega}) < 0 \end{cases}$$

DLMF 10.27.9

DLMF 10.27.8

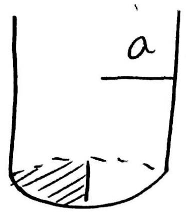
$$\boxed{g(\bar{r}) = \begin{cases} i/4\, H_0^{(1)}(|\tilde{\omega}| r) & \mathrm{sgn}(\tilde{\omega}) > 0 \\ -i/4\, H_0^{(2)}(|\tilde{\omega}| r) & \mathrm{sgn}(\tilde{\omega}) < 0 \end{cases}}$$

$$H_0^{(1,2)} \sim J_0 \pm i N_0$$

$$g(r \to 0) = -\frac{1}{4} N_0(|\tilde{\omega}| r) \to -\frac{1}{2\pi}\ln(r)$$

$$g(r) e^{-i\omega t} \xrightarrow[r\to\infty]{} \begin{cases} \frac{1}{\sqrt{r}}\, e^{i\frac{|\omega|}{c}(r - ct)}\, e^{-i\pi/4} & \omega > 0 \\ \frac{1}{\sqrt{r}}\, e^{-i\frac{|\omega|}{c}(r - ct)}\, e^{+i\pi/4} & \omega < 0 \end{cases}$$

(16)



$$u(r,\varphi) = \frac{i V_0}{2} g(r,\varphi)\, e^{-i\Omega t} + c.c.$$

$$g(r,\varphi) = \begin{cases} 1 & \varphi \in [0, \pi) \\ -1 & \varphi \in (\pi, 2\pi] \end{cases}$$

$$\nabla^2 \psi = \left[\left(\frac{\omega}{c}\right)^2 - k_{nm}^2\right] \psi$$

$$\psi(r,\varphi) = \frac{e^{im\varphi}}{\sqrt{2\pi}} J_m(k_n r) \qquad J'_m(k_n a) = 0$$

$$\partial_z^2 C_n(z) = \underbrace{\left[\left(\frac{\omega}{c}\right)^2 - k_{nm}^2\right]}_{\Omega} C_{nm}(z)$$

$$C_n(z) \begin{cases} \mathrm{Sen}(\Omega z) & (k_{nm} < \omega/c) \\ \mathrm{Senh}(\Omega z) & (k_{nm} > \omega/c) \end{cases}$$

$$\rho_0\, \partial_t^2 u(r,\varphi,t) = (-\partial_z \psi)(r,\varphi,z=0,t)$$

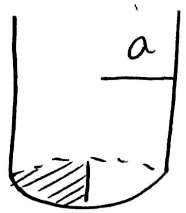
$$-\frac{i \rho V_0}{2} g(r,\varphi)\, \omega^2 = \Omega\, C_n(z)\, \phi_{nm}(r,\varphi)$$

$$D_{nm} = \frac{\langle \phi(r,\varphi) | g_m \rangle}{b_{nm}} \;;\; b_{nm} = \int_0^a r\, dr \left[J_m(k_{nm} r)\right]^2$$

$$C_{nm} = \frac{\rho_0\, \omega^2 u_0 D_{nm}}{2\Omega}$$

$$g_m = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} e^{im\varphi} g(r,\varphi) = -\frac{4i}{m\sqrt{2\pi}} \qquad m = 1, 3, 5$$

16



$$u(r,\varphi) = \frac{i V_0}{2} g(r,\varphi) e^{-i\Omega t} + c.c.$$

$$g(r,\varphi) = \begin{cases} 1 & \varphi \in [0, \pi) \\ -1 & \varphi \in (\pi, 2\pi] \end{cases}$$

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$$\psi(r,\varphi) = \frac{e^{im\varphi}}{\sqrt{2\pi}} J_m(k_n r) \qquad J_m'(k_n a) = 0$$

$$\partial_z^2 C_n(z) = \underbrace{\left[\left(\frac{\omega}{c}\right)^2 - k_{nm}^2\right]}_{\Omega} C_{nm}(z)$$

$$C_n(z) \begin{cases} \operatorname{Sen}(\Omega z) & (k_{nm} < \omega/c) \\ \operatorname{Senh}(\Omega z) & (k_{nm} > \omega/c) \end{cases}$$

$$\rho_0 \partial_t^2 u(r,\varphi,t) = (-\partial_z \psi)(r,\varphi,z=0,t)$$

$$-\frac{i \rho V_0}{2} g(r,\varphi)\, \omega^2 = \Omega^2 C_n(z)\, \phi_{nm}(r,\varphi)$$

$$D_{nm} = \frac{\langle \phi(r,\varphi) | g_m \rangle}{b_{nm}} \; ; \; b_{nm} = \int_0^a r\,dr \left[J_m(k_{nm} r)\right]^2$$

$$C_{nm} = \frac{\rho_0 \omega^2 u_0 D_{nm}}{2\Omega}$$

$$g_m = \frac{1}{\sqrt{2\pi}} \int_0^{2\pi} e^{im\varphi} g(r,\varphi) = -\frac{4i}{m\sqrt{2\pi}} \qquad m = 1, 3, 5$$

13 cont

T_i $\quad T(\bar r, t) = T_0 + \tilde T(\bar r, t)$

T_0 $\quad \tilde T \propto e^{-\lambda_n t} \quad t \to \infty \quad \tilde T = 0 \quad T(r, \infty) = T_0$

m=0 → invariante frente a traslaciones

$$\cancel{\nabla^2 \tilde T(\bar r, t)} = (\partial_t - \chi \nabla_r^2)\tilde T(\bar r, t) = 0$$

$$\tilde T(\bar r, t) = \underbrace{\psi(\bar r, t)} C_n(t)$$ ← apartado anterior

$\chi k_n^2 = \lambda_n$ $\quad \partial_t C_n(t) = \chi C_n(t) k_n^2$

$$C_n(t) = C_n(0) e^{-\lambda_n t} \qquad C_n(0) = \langle \psi | T_i \rangle$$

$$\psi_{m=0}(r, \varphi) = \sum_n \sqrt{\frac{2}{b}} \, \mathrm{Sen}\left(\frac{n\pi}{b} z\right) \frac{1}{\sqrt{2\pi}} J_{0n}(k_{0n} r)$$

$$\langle \tilde T | T_i' \rangle = \frac{T_i'}{\sqrt{b\pi}} \int_0^b dz \, \mathrm{Sen}\left(\frac{n\pi}{b} z\right) \int_0^r r J_{0n}(k_{0n} r) \int_0^{2\pi} d\varphi \, b_{nm}^{-1}$$

$T_i' = (T_i - T_0)$

n = impar $\qquad k_{0n} r = x \qquad dr = \frac{dx}{k_{0n}}$

$$= \frac{T_i'}{\sqrt{b\pi}} \frac{b}{n\pi} 2 \cdot 2\pi \int_0^{k_0 a} \frac{x J_0(x)\, dx}{b_{nm}} = \frac{T_i' b}{\sqrt{b\pi}\, n\pi} 2 Ka J_1(Ka) 2\pi b_{nm}^{-1}$$

$$C_n(0) = \sqrt{\frac{b}{\pi}} \frac{T_i'}{n\pi} 2 k_0 a J_1(k_{n0} a) 2\pi b_{nm}^{-1}$$

$$T(\bar r, t) = \sum \psi_{m=0}(\bar r, t) C_n(0) e^{-\lambda_n t} + T_0$$

$$\boxed{T(\bar r, t) = T_0 + \sum_n \frac{T_i'}{n} \mathrm{Sen}\left(\frac{n\pi}{b} z\right) \frac{4}{\pi} \frac{J_{0n}}{b_{nm}} (k_n a) J_1(k_n a) e^{-\lambda_n t} k_n a}$$

falta normalizar C_{nm} (sacar b_{nm}^{-1})

$$b_{nm} = \int_0^a r\,dr \left[J_m(k_{nm} r)\right]^2 = \frac{a^2}{2} J_1^2(k_n a)$$

$x(t)\, p(r) \qquad p(r)\dot x(t) - \chi \ddot p(r)\, x(t)$

(13) cont

T_i

T_0

$$T(\bar r, t) = T_0 + \tilde T(\bar r, t)$$

$$\tilde T \propto e^{-\lambda_n t} \quad t \to \infty \quad \tilde T = 0 \quad T(r, \infty) = T_0$$

$m=0 \to$ invariante frente a traslaciones

$$\cancel{\nabla^2 \tilde T(\bar r, t)} = \quad (\partial_t - \chi \nabla_r^2)\tilde T(\bar r, t) = 0$$

$$\tilde T(\bar r, t) = \underbrace{\psi(\bar r, t)}_{\text{apartado anterior}} C_n(t)$$

$$\chi K_n^2 = \lambda_n \qquad \partial_t C_n(t) = \chi C_n(t) K_n^2$$

$$C_n(t) = C_n(0) e^{-\lambda_n t} \qquad C_n(0) = \langle \psi | T_i \rangle$$

$$\psi_{m=0}(r, \varphi) = \sum_n \sqrt{\frac{2}{b}} \operatorname{Sen}\left(\frac{n\pi}{b} z\right) \frac{1}{\sqrt{2\pi}} J_{0n}(K_{0n} r)$$

$$\langle \tilde T | T_i' \rangle = \frac{T_i'}{\sqrt{b\pi}} \int_0^b dz \operatorname{Sen}\left(\frac{n\pi}{b} z\right) \int_0^r r J_{0n}(K_{0n} r) \int_0^{2\pi} d\varphi \; b_{nm}^{-1}$$

$T_i' = (T_i - T_0)$

$n =$ impar

$$K_{0n} r = x \qquad dr = \frac{dx}{K_{0n}}$$

$$= \frac{T_i'}{\sqrt{b\pi}} \frac{b}{n\pi} 2 \cdot 2\pi \int_0^{K_0 a} \frac{x J_0(x)\, dx}{b_{nm}} = \frac{T_i' b}{\sqrt{b\pi}\, n\pi} 2 K a J_1(Ka) \sqrt{\ } 2\pi b_{nm}^{-1}$$

$$C_n(0) = \sqrt{\frac{b}{\pi}} \frac{T_i'}{n\pi} 2 K_{0} a J_1(K_{n0} a) 2\pi\, b_{nm}^{-1}$$

$$T(\bar r, t) = \sum \psi_{m=0}(\bar r, t) C_n(0) e^{-\lambda_n t} + T_0$$

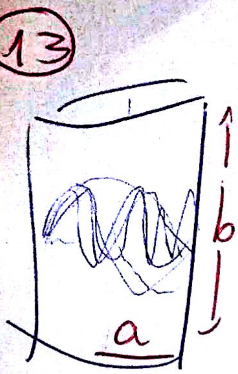
$$\boxed{T(\bar r, t) = T_0 + \sum_n \frac{T_i'}{n} \operatorname{Sen}\left(\frac{n\pi}{b} z\right) \frac{4}{\pi} \frac{J_{0n}}{b_{nm}} (K_n a) J_1(K_n A) e^{-\lambda_n t} K_n a}$$

falta normalizar C_{nm} (sacar b_{nm}^{-1})

$$b_{nm} = \int_0^a r\,dr \left[J_m(K_{nm} r)\right]^2 = \frac{a^2}{2} J_1^2(K_n a)$$

$x(t) \, p(r) \qquad p(r) \dot x(t) - \chi \ddot p(r) x(t)$

13



$V(z=0)=V(z=b)=V(r=a)=0$

$\nabla^2\psi=0 \Rightarrow$ autovalores son 0

a)

$\nabla^2=\Delta_2+\partial^2_z$

$\psi(r,\varphi,z)=\sum_{n=-\infty}^{\infty}\sum_{1}^{\infty}\phi_{nm}(r,\varphi)\,C_{nm}(z)$

• $\phi_{nm}(\bar r) \rightarrow -\nabla^2\phi_{nm}(\bar r)=k^2\phi_{nm}(\bar r)$

$\cancel{\phi_{nm}(\bar r)=\frac{1}{\sqrt{2\pi}}e^{im\varphi}J_m(k_{nm}r)}$

$C_{nm}(z)$ $\quad C_{nm}(0)=0$, $\quad C_{nm}(b)=0$ $\quad \sqrt{\frac{2}{b}}\,\mathrm{Sen}(k_n z)$ $\quad k_n=\frac{n\pi}{b}$

$\phi_{nm}(r,\varphi)=R_n(r)\,g_n(\varphi)$ $\qquad \nabla^2(r,\varphi)=\underbrace{\partial^2_r+\frac{1}{r}\partial_r+\frac{1}{r^2}\partial^2_\varphi}$

$$\begin{cases} g''_m(\varphi)=-m^2 g_m(\varphi) \\ R''_m(r)+\frac{1}{r}R'_m(r)-\frac{m^2}{r^2}R_m(r)=0 \end{cases}$$

$g_m(\varphi)\propto e^{im\varphi} \Rightarrow g_m(\varphi)=\frac{1}{\sqrt{2\pi}}e^{im\varphi}$

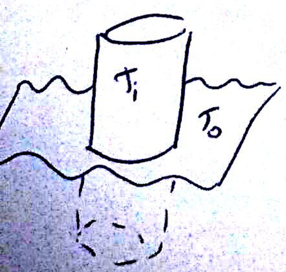
$R_m(r)=a_m r^{|m|}+b_m r^{-|m|}$ $\quad \forall m\neq 0$

$m=0 \rightarrow a_0+\ln r$

$\psi(r,\varphi,z)=\phi\, R_m(r)\,g_m(\varphi)\,C_{nm}(z)$

$$\boxed{\psi(r,\varphi,z)=\sum_{m,n}\sqrt{\frac{2}{b}}\,\mathrm{Sen}\left(\frac{n\pi}{b}z\right)\left[\frac{1}{\sqrt{2\pi}}e^{im\varphi}\underbrace{\left(a_m r^{|m|}+b_m r^{-|m|}\right)+a_0+\ln r}_{\text{(regular en } r=0) \rightarrow J_{mn}(k_{mn}r)}\right]}$$

b)

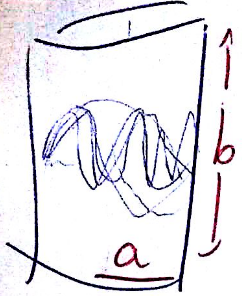


$\tilde T=T_0+\tilde T_1$

$\tilde T(t=0)=T_0$

13

$V(z=0)=V(z=b)=V(r=a)=0$



a) $\nabla^2\psi=0 \Rightarrow$ autovalores son 0

$\nabla^2 = \Delta_2 + \partial^2_z$

$$\psi(r,\varphi,z)=\sum_{n=-\infty}^{\infty}\sum_{1}^{\infty}\phi_{nm}(r,\varphi)\,C_{nm}(z)$$

• $\phi_{nm}(\bar{r}) \rightarrow -\nabla^2\phi_{nm}(\bar{r})=k^2\phi_{nm}(\bar{r})$

$\cancel{\phi_{nm}(\bar{r})=\frac{1}{\sqrt{2\pi}}e^{im\varphi}J_m(k_{mn}r)}$

$C_{nm}(z)$ $\quad C_{nm}(0)=0$, $\quad C_{nm}(b)=0$ $\quad \sqrt{\frac{2}{b}}\,\text{Sen}(k_n z)$ $\quad k_n=\frac{n\pi}{b}$

$\phi_{nm}(r,\varphi)=R_n(r)\,g_n(\varphi)$ $\qquad \nabla^2(r,\varphi)=\underbrace{\partial^2_r+\frac{1}{r}\partial_r+\frac{1}{r^2}\partial^2_\varphi}$

$$\begin{cases} g''_m(\varphi)=-m^2 g_m(\varphi) \\ R''_m(r)+\frac{1}{r}R'_m(r)-\frac{m^2}{r^2}R_m(r)=0 \end{cases}$$

$g_m(\varphi)\propto e^{im\varphi} \Rightarrow g_m(\varphi)=\frac{1}{\sqrt{2\pi}}e^{im\varphi}$

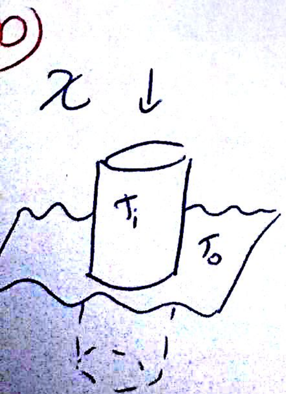
$R_m(r)=a_m r^{|m|}+b_m r^{-|m|}$ $\quad \forall m\neq 0$

$m=0 \rightarrow a_0+\ln r$

$\psi(r,\varphi,z)=\sum R_m(r)\,g_m(\varphi)\,C_{nm}(z)$

$$\boxed{\psi(r,\varphi,z)=\sum_{m,n}\sqrt{\frac{2}{b}}\,\text{Sen}\left(\frac{n\pi}{b}z\right)\left[\frac{1}{\sqrt{2\pi}}e^{im\varphi}\underbrace{\left(a_m r^{|m|}+b_m r^{-|m|}\right)+a_0+\ln r}_{\text{(regular en } r=0) \rightarrow J_{mn}(k_{mn}r)}\right]}$$

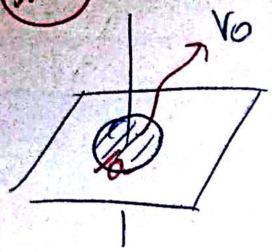
b)



$\tilde{T}=T_0+\tilde{T}_L$

$\tilde{T}(t=0)=T_0$

12



probar $V(r,z)=V_0 a\int_0^\infty dk\, J_1(ka)\, J_0(kr)\, e^{-kz}$

$$V=\begin{cases} V_0 & r<a \\ 0 & r>a \end{cases}$$

como no depende de $\varphi \Rightarrow \underline{m=0} \rightarrow$ invariante frente a rotaciones

$\nabla^2\psi(r,z)=0$ $\qquad \partial_z^2 C_m(k,z)=k^2 C_m(k,z)$

$$\boxed{\psi(r,\varphi,z)=\sum_{-\infty}^{\infty}\int_0^\infty k\,dk\, C_m(kz)\,\phi(r,\varphi)}$$

$$C_m(k,z)=\begin{cases} C_m(k)e^{-kz} & (z>0) \\ \underbrace{C_m(k)}_{V_m(k)}e^{kz} & (z<0)\end{cases}$$

sólo queremos $z>0$

$$\phi_{km}(r,\varphi)=\frac{e^{im\varphi}}{\sqrt{2\pi}}J_m(kr)$$

$$V_m(k)=\langle\phi_{km}|V\rangle=\int_0^a r\,dr\int_0^{2\pi}d\varphi\;\phi^*_{km}(r,\varphi)\,\overset{V_0}{V(r,\varphi)}$$

$m=0$

$$=\sqrt{2\pi}\,V_0\int_0^a r\,dr\,J_0(kr)=\sqrt{2\pi}\,\frac{V_0}{k^2}\int_0^{ka}x\,dx\,J_0(x)=\sqrt{2\pi}\,\frac{aV_0}{k}J_1(ka)$$

tablas

$(x=kr \quad dx=k\,dr)$

$$\boxed{\psi(r,\varphi)=\int_0^\infty V_0 a\, e^{-kz}J_1(ka)\,J_0(kr)}$$

$$\begin{cases} J_m(0)=\frac{1}{m!}\left(\frac{x}{2}\right)^m \\ J_0(0)=1 \end{cases}$$

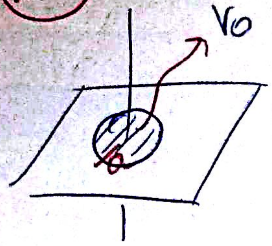
• $r=0$

$$\psi(r=0,z)=\int_0^\infty V_0 a\, e^{-kz}J_1(ka)\,J_0(k\cdot 0)\,dk=$$

tablas

$$\boxed{\psi(0,z)=V_0\left[1-\frac{z}{(z^2+a^2)^{1/2}}\right]}$$

12



probar $V(r,z) = V_0 a \int_0^\infty dk\, J_1(ka) J_0(kr) e^{-kz}$

$$V = \begin{cases} V_0 & r < a \\ 0 & r > a \end{cases}$$

como no depende de $\varphi \Rightarrow \underline{m=0} \rightarrow$ invariante frente a rotaciones

$\nabla^2 \psi(r,z) = 0 \qquad \partial_z^2 C_m(k,z) = k^2 C_m(k,z)$

$$\boxed{\psi(r,\varphi,z) = \sum_{-\infty}^{\infty} \int_0^\infty k\,dk\, C_m(kz)\,\phi(r,\varphi)}$$

$$C_m(k,z) = \begin{cases} C_m(k)\, e^{-kz} & (z>0) \\ C_m(k)\, e^{kz} & (z<0) \end{cases}$$

sólo queremos $z>0$ $\qquad C_m(k) \hookrightarrow V_m(k)$

$$\phi_{km}(r,\varphi) = \frac{e^{im\varphi}}{\sqrt{2\pi}} J_m(kr)$$

$$V_m(k) = \langle \phi_{km} | V \rangle = \int_0^a r\,dr \int_0^{2\pi} d\varphi\, \phi^*_{km}(r,\varphi)\, V(r,\varphi) =$$

($V(r,\varphi) \to V_0$)

$m=0$

$$= \sqrt{2\pi}\, V_0 \int_0^a r\,dr\, J_0(kr) = \sqrt{2\pi}\, \frac{V_0}{k^2} \int_0^{ka} x\,dx\, J_0(x) = \sqrt{2\pi}\, \frac{a V_0}{k} J_1(ka) \ \ldots$$

($x = kr \quad dx = k\,dr$) $\qquad$ tablas

$$\boxed{\psi(r,\varphi) = \int_0^\infty V_0 a\, e^{-kz} J_1(ka) J_0(kr)}$$

$$\begin{cases} J_m(0) = \frac{1}{m!}\left(\frac{x}{2}\right)^m \\ J_0(0) = 1 \end{cases}$$

• $r=0$

$$\psi(r=0,z) = \int_0^\infty V_0 a\, e^{-kz} J_1(ka) J_0(k\cdot 0)\, dk =$$

tablas

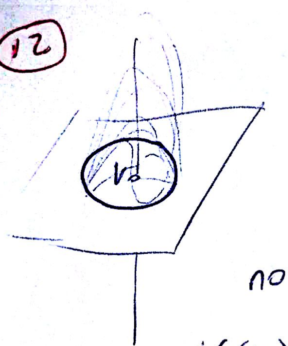
$$\boxed{\psi(0,z) = V_0\left[1 - \frac{z}{(z^2+a^2)^{1/2}}\right]}$$

$x_{n=6} \approx 18'1 \qquad x_{n=?} \approx 24'4$

$$V(r,z) = V_0 a \int_0^\infty dk\, J_1(ka)\, J_0(kr)\, e^{-kz}$$

$x = kr$
$dx = k\,dr$

(12)



$V_1(r>a, z=0) = 0$

$V_2(r<a, z=0) = V_0$

invariante frente a rotaciones

no depende de $\varphi \to V_{m=0}(k)$

$$V_m(k) = \int_0^r dr \int_0^{2\pi} d\varphi \frac{e^{-im\varphi}}{\sqrt{2\pi}} J_m(kr)\, V(r,\varphi) \qquad V(r,\varphi) \to V_0$$

tablas

$m=0$

$$V_0(k) = \sqrt{2\pi} \int_0^r r\,dr\, J_0(kr) = \sqrt{2\pi} \frac{V_0}{k^2} \int_0^{ka} x\,dx\, J_0(x) = \sqrt{2\pi} \frac{V_0}{k} a\, J_1(kb)$$

$$\boxed{\Psi_m(r,z) = \sum_{-\infty}^{\infty} \int_0^\infty k\,dk\, V_m(k)\, e^{-kz}} \qquad \boxed{\phi_{k_m}(r) = V_0 a \int_0^\infty dk\, J_1(ka)\, J_0(kr)\, e^{-kz}}$$

en $r=0$ (eje z)

$$V(r=0,z) = V_0 a \int_0^\infty dk\, e^{-kz} J_1(kb) = V_0\left[1 - \frac{z}{\sqrt{z^2+a^2}}\right]$$

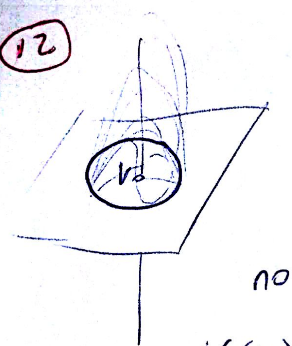
$x_{n=6} \approx 18'1$ $\qquad x_{n=7} \approx 24'4$

$$V(r,z) = V_0 a \int_0^\infty dk\, J_1(ka)\, J_0(kr)\, e^{-kz}$$

$x = kr$

$dx = k\,dr$

12



$V_1(r>a, z=0) = 0$

$V_2(r<a, z=0) = V_0$

invariante frente a rotaciones

no depende de $\varphi \to V_{m=0}(k)$

$$V_m(k) = \int_0^r dr \int_0^{2\pi} d\varphi\, \frac{e^{-im\varphi}}{\sqrt{2\pi}}\, J_m(kr)\, V(r,\varphi) \quad V(r,\varphi) \to V_0$$

tablas

$m=0$

$$V_0(k) = \sqrt{2\pi}\int_0^r r\,dr\, J_0(kr) = \sqrt{2\pi}\,\frac{V_0}{k^2}\int_0^{ka} x\,dx\, J_0(x) = \sqrt{2\pi}\,\frac{V_0}{k}\,a\, J_1(kb)$$

$$\boxed{\Psi_m(r,z) = \sum_{-\infty}^{\infty}\int_0^\infty k\,dk\, V_m(k)\, e^{-kz}\, \phi_{k_m}(r)} = V_0 a \int_0^\infty dk\, J_1(ka)\, J_0(kr)\, e^{-kz}$$

en $r=0$ (eje z)

$$V(r=0, z) = V_0 a \int_0^\infty dk\, e^{-kz}\, J_1(kb) = V_0\left[1 - \frac{z}{\sqrt{z^2+a^2}}\right]$$

$$J_{m-1}(x) = \left(\frac{x}{2}\right)^{m-1} \sum_{j=0}^{\infty} \frac{(-1)^j}{j!(m-1+j)!}\left(\frac{x}{2}\right)^{2j} = \sum_{j=1}^{\infty}\frac{(-1)^j}{j!\,(m-1+j)!}\left(\frac{x}{2}\right)^{2j+m-1}$$

$$J_m'(x) = \frac{m}{x}J_m(x) + \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!}\, 2j\, \frac{x^{2j+m-1}}{2^{2j+m}} = J_{m-1}(x) - \frac{m}{x}J_m(x)$$

$$J_{m-1}(x) = 2\frac{m}{x}J_m(x) + \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!}\,\frac{x^{2j+m-1}}{2^{2j+m}}\, 2j =$$

$$= 2\frac{m}{x}J_m(x) + \sum_{j=1}^{\infty} \frac{(-1)^j}{(j-1)!\,(m+j)!}\left(\frac{x}{2}\right)^{2j+m-1}$$

$$J_{m-1}(x) = \frac{2}{x}\left(\frac{x}{2}\right)^m \sum_{j=1}^{\infty} \frac{(-1)^j}{j!\,(m-1+j)!}\left(\frac{x}{2}\right)^{2j} = \cancel{\frac{2}{x}\left(\frac{x}{2}\right)^m \sum_{j=2}^{\infty}}$$

$j \to j+1$

$$\cancel{\frac{(-1)^{j+1}}{(j+1)!\,(m+j)!}\left(\frac{x}{2}\right)^{2j+2}} = \cancel{\frac{2}{x}\left(\frac{x}{2}\right)^m \sum_{j=2}^{\infty} \frac{(-1)^{j+1}(m+j)}{(j+1)!\,(m-1+j)!}\left(\frac{x}{2}\right)^{2j+2}}$$

$$= \cancel{2\frac{m}{x}\left(\frac{x}{2}\right)^m \sum_{j=2}^{\infty} \frac{(-1)^{j+1}}{(j+1)!\,(m-1+j)!}\left(\frac{x}{2}\right)^{2j+2}}$$

$$= \sum_{j=1}^{\infty} \frac{(-1)^j}{j!\,(m-1+j)!}\, 2\, \frac{x^{2j+m-1}}{2^{2j+m}} = \sum_{j=1}^{\infty} \frac{(-1)^j (j+m)\, 2}{j!\,(m+j)!}\,\frac{x^{2j+m-1}}{2^{2j+m}} =$$

$$= \frac{m}{x}J_m(x) + \underbrace{\sum_{j=1}^{\infty} \frac{(-1)^j (2j+m)}{j!\,(m+j)!}\,\frac{x^{2j+m-1}}{2^{2j+m}}}_{J_m'(x)}$$

$$J_{m-1}(x) + J_{m+1}(x) = \frac{m}{x}J_m(x) + \cancel{J_m'(x)} + \frac{m}{x}J_m(x) + -\cancel{J_m'(x)}$$

$$= \boxed{\frac{2m}{x}J_m(x)}$$

$$J_{m-1}(x) = \left(\frac{x}{2}\right)^{m-1} \sum_{j=0}^{\infty} \frac{(-1)^j}{j!\,(m-1+j)!}\left(\frac{x}{2}\right)^{2j} = \sum_{j=1}^{\infty}\frac{(-1)^j}{j!\,(m-1+j)!}\left(\frac{x}{2}\right)^{2j+m-1}$$

$$J_m'(x) = \frac{m}{x}J_m(x) + \sum_{j=1}^{\infty} \frac{(-1)^j}{j!\,(m+j)!}\, 2j\, \frac{x^{2j+m-1}}{2^{2j+m}} = J_{m-1}(x) - \frac{m}{x}J_m(x)$$

$$J_{m-1}(x) = 2\frac{m}{x}J_m(x) + \sum_{j=1}^{\infty} \frac{(-1)^j}{j!\,(m+j)!}\,\frac{x^{2j+m-1}}{2^{2j+m}}\, 2j =$$

$$= 2\frac{m}{x}J_m(x) + \sum_{j=1}^{\infty} \frac{(-1)^j}{(j-1)!\,(m+j)!}\left(\frac{x}{2}\right)^{2j+m-1}$$

$$J_{m-1}(x) = \frac{2}{x}\left(\frac{x}{2}\right)^m \sum_{j=1}^{\infty} \frac{(-1)^j}{j!\,(m-1+j)!}\left(\frac{x}{2}\right)^{2j} = \cancel{\frac{2}{x}\left(\frac{x}{2}\right)^m \sum_{j=2}^{\infty}}$$

$j \to j+1$

$$\cancel{\frac{(-1)^{j+1}}{(j+1)!\,(m+j)!}\left(\frac{x}{2}\right)^{2j+2}} = \cancel{\frac{2}{x}\left(\frac{x}{2}\right)^m \sum_{j=2}^{\infty} \frac{(-1)^{j+1}(m+j)}{(j+1)!\,(m+1+j)!}\left(\frac{x}{2}\right)^{2j+2}}$$

$$= \cancel{2\frac{m}{x}\left(\frac{x}{2}\right)^m \sum_{j=2}^{\infty} \frac{(-1)^{j+1}}{(j+1)!\,(m+1+j)!}\left(\frac{x}{2}\right)^{2j+2}}$$

$$= \sum_{j=1}^{\infty} \frac{(-1)^j}{j!\,(m-1+j)!}\, 2\, \frac{x^{2j+m-1}}{2^{2j+m}} = \sum_{j=1}^{\infty} \frac{(-1)^j (j+m)\, 2}{j!\,(m+j)!}\, \frac{x^{2j+m-1}}{2^{2j+m}} =$$

$$= \frac{m}{x}J_m(x) + \sum_{j=1}^{\infty} \underbrace{\frac{(-1)^j (2j+m)}{j!\,(m+j)!}\, \frac{x^{2j+m-1}}{2^{2j+m}}}_{J_m'(x)}$$

$$J_{m-1}(x) + J_{m+1}(x) = \frac{m}{x}J_m(x) + \cancel{J_m'(x)} + \frac{m}{x}J_m(x) + -\cancel{J_m'(x)}$$

$$= \boxed{\frac{2m}{x}J_m(x)}$$

(14)

Hoja 4 HMIII

$$\begin{cases} J_{m+1}(x) = \frac{m}{x} J_m(x) - J_m'(x) \\ J_{m-1}(x) = \frac{m}{x} J_m(x) + J_m'(x) \end{cases}$$

1)

$$J_m(x) = \left(\frac{x}{2}\right)^m \sum_{j=0}^{\infty} \frac{(-1)^j}{j!(m+j)!} \left(\frac{x}{2}\right)^{2j}$$

$$J_{m\pm1}(x) = J_m(x) \frac{m}{x} \mp J_m'(x)$$

$$-J_m'(x) = \frac{\partial J_m(x)}{\partial x} = \left[\sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!} (m+2j) \frac{x^{2j+m-1}}{2^{2j+m}}\right] J_m'(x)$$

$$-\mp J_m'(x) = J_{m\pm1}(x) - J_m(x)\frac{m}{x}$$

$$J_{m+1}(x) = \left(\frac{x}{2}\right)^{m+1} \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+1+j)!} \left(\frac{x}{2}\right)^{2j} = \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+1+j)!} \left(\frac{x}{2}\right)^{2j+m+1}$$

$$J_m(x)\frac{m}{x} = \frac{m}{x}\left(\frac{x}{2}\right)^m \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!} \left(\frac{x}{2}\right)^{2j} = \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!} \left(\frac{x}{2}\right)^{2j+m-1} \frac{m}{2}$$

$$J_{m+1}(x) + J_m(x) = \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+1+j)!} \left(\frac{x}{2}\right)^{2j+m+1} - \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!} \left(\frac{x}{2}\right)^{2j+m-1} \frac{m}{2}$$

$$J_m'(x) = \underbrace{\frac{m}{x} \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!} \left(\frac{x}{2}\right)^{2j+m}}_{\frac{m}{x} J_m(x)} + \underbrace{\sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!}\, 2j\, \frac{x^{2j+m-1}}{2^{2j+m}}}_{\text{tiene que ser } -J_{m+1}(x)}$$

$$\sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!}\, 2j\, \frac{x^{2j+m-1}}{2^{2j+m}} \xRightarrow{j \to j+1} \sum_{j=0}^{\infty} \frac{(-1)^{j+1}}{(j+1)!(m+1+j)!} (2j+2) \frac{x^{2j+m+1}}{2^{2j+m+2}} =$$

$$= -\sum_{j=0}^{\infty} \frac{(-1)^j}{j!(m+1+j)!} \left(\frac{x}{2}\right)^{2j+m+1} \quad //$$

(14)

Hoja 4 HMIII

$$\begin{cases} J_{m+1}(x) = \frac{m}{x} J_m(x) - J_m'(x) \\ J_{m-1}(x) = \frac{m}{x} J_m(x) + J_m'(x) \end{cases}$$

(1)

$$J_m(x) = \left(\frac{x}{2}\right)^m \sum_{j=0}^{\infty} \frac{(-1)^j}{j!(m+j)!} \left(\frac{x}{2}\right)^{2j}$$

$$J_{m\pm1}(x) = J_m(x)\frac{m}{x} \mp J_m'(x)$$

$$-J_m'(x) = \frac{\partial J_m(x)}{\partial x} = \left[\sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!} (m+2j) \frac{x^{2j+m-1}}{2^{2j+m}}\right] J_m'(x)$$

$$-\mp J_m'(x) = J_{m\pm1}(x) - J_m(x)\frac{m}{x}$$

$$J_{m+1}(x) = \left(\frac{x}{2}\right)^{m+1} \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+1+j)!} \left(\frac{x}{2}\right)^{2j} = \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+1+j)!} \left(\frac{x}{2}\right)^{2j+m+1}$$

$$J_m(x)\frac{m}{x} = \frac{m}{x}\left(\frac{x}{2}\right)^m \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!} \left(\frac{x}{2}\right)^{2j} = \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!} \left(\frac{x}{2}\right)^{2j+m-1} \frac{m}{2}$$

$$J_{m+1}(x) + J_m(x) = \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+1+j)!} \left(\frac{x}{2}\right)^{2j+m+1} - \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!} \left(\frac{x}{2}\right)^{2j+m-1} \frac{m}{2}$$

$$J_m'(x) = \underbrace{\frac{m}{x} \sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!} \left(\frac{x}{2}\right)^{2j+m}}_{\frac{m}{x} J_m(x)} + \underbrace{\sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!}\, 2j\, \frac{x^{2j+m-1}}{2^{2j+m}}}_{\text{tiene que ser } -J_{m+1}(x)}$$

$j \to j+1$

$$\sum_{j=1}^{\infty} \frac{(-1)^j}{j!(m+j)!}\, 2j\, \frac{x^{2j+m-1}}{2^{2j+m}} \Rightarrow \sum_{j=0}^{\infty} \frac{(-1)^{j+1}}{(j+1)!(m+1+j)!} (2j+2) \frac{x^{2j+m+1}}{2^{2j+m+2}} =$$

$$= -\sum_{j=0}^{\infty} \frac{(-1)^j}{j!(m+1+j)!} \left(\frac{x}{2}\right)^{2j+m+1} \quad //$$

(5)

$$|k,m\rangle \longrightarrow \psi_{k,m}(\vec r) = \frac{e^{im\varphi}}{\sqrt{2\pi}} J_m(kr)$$

$$|\vec k\rangle \Rightarrow \phi_{\vec k}(\vec r) = \frac{1}{2\pi} e^{i\vec k\vec r} \qquad \langle \vec k | k', m\rangle = C_m(k)\,\delta(k-k')\,e^{im\varphi_{\vec k}}$$

$$|k,m\rangle = \int d^3k \langle \vec k | k', m\rangle |\vec k\rangle ;$$

$$\frac{1}{\sqrt{2\pi}} J_m(kr) = V\, C_m(k) \int \frac{d\varphi'}{2\pi} e^{ikr\cos\varphi'} e^{im\varphi'} \quad ; \lim_{r\to 0} J_m(kr) = \left(\frac{kr}{2}\right)^m \frac{1}{m!}$$

$$e^{ikr\cos(\varphi-\varphi_k)} = \sum_m i^m e^{-im(\varphi_k-\varphi)} J_m(kr)$$

$$\lim_{r\to 0} \int \frac{d\varphi'}{\sqrt{2\pi}} e^{ikr\cos\varphi'} e^{im\varphi'} = \left(\frac{ikr}{2}\right)^m \frac{1}{m!}$$

$$\downarrow$$

$$e^{i\vec k\vec r} = \sum_m i^m J_m(kr)\, e^{im(\varphi-\varphi_k)}$$

$$\downarrow$$

$$\boxed{C_m(k) = \frac{(-i)^m}{k} \frac{1}{\sqrt{2\pi}}}$$

3d

$$|k,\ell,m\rangle \longrightarrow \psi_{k\ell m}(\vec r) = j_\ell(kr)\, Y_{\ell m}(\theta,\varphi)$$

$$|\vec k\rangle \longrightarrow \phi_{\vec k}(\vec r) = \frac{1}{(2\pi)^{3/2}} e^{i\vec k\vec r}$$

$$\langle \vec k | k';\ell m\rangle \propto \delta(k-k')\, Y_{\ell m}(\theta_{\vec k}, \varphi_{\vec k})$$

$$e^{i\vec k\vec r} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} C_\ell(k)\, Y_{\ell m}(\theta_k, \varphi_k)\, j_\ell(kr)\, Y_{\ell m}(\theta,\varphi)$$

$$C_\ell(k) \frac{2\ell+1}{2} j_\ell(kr) = \frac{2\ell+1}{2} \int_{-1}^{1} dx\, e^{ikrx} P_\ell(x)$$

$$\vec r \to 0 ; \; j_\ell(kr) \to \frac{(kr)^\ell}{(2\ell+1)!!}$$

$$\lim_{r\to 0} \int dx\, e^{ikrx} P_\ell(x) = \frac{(ikr)^\ell}{(2\ell+1)!!} 2$$

$$\boxed{C_\ell(k) = 4\pi i^\ell} \qquad e^{i\vec k\vec r} = 4\pi \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} i^\ell\, Y^*_{\ell m}(\theta_{\vec k}, \varphi_{\vec k})\, j_\ell(kr)\, Y_{\ell m}(\theta,\varphi)$$

modulo

$$\downarrow$$

$$= 4\pi \sum_{\ell=0}^{\infty} i^\ell (2\ell+1)\, P_\ell(\cos\gamma_{\vec r\vec k})\, j_\ell(kr)$$

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$$|k,m\rangle \longrightarrow \psi_{k,m}(\vec r) = \frac{e^{im\varphi}}{\sqrt{2\pi}} J_m(kr)$$

$$|\vec k\rangle \Rightarrow \phi_{\vec k}(\vec r) = \frac{1}{2\pi} e^{i\vec k\vec r} \qquad \langle \vec k | k', m\rangle = C_m(k)\,\delta(k-k')\,e^{im\varphi_{\vec k}}$$

$$|k,m\rangle = \int d^3k \langle \vec k | k', m\rangle |\vec k\rangle ;$$

$$\frac{1}{\sqrt{2\pi}} J_m(kr) = V\, C_m(k) \int \frac{d\varphi'}{2\pi} e^{ikr\cos\varphi'} e^{im\varphi'} \quad ; \quad \lim_{r\to 0} J_m(kr) = \left(\frac{kr}{2}\right)^m \frac{1}{m!}$$

$$e^{ikr\cos(\varphi-\varphi_k)} = \sum_m i^m e^{-im(\varphi_k-\varphi)} J_m(kr)$$

$$\downarrow$$

$$e^{i\vec k\vec r} = \sum_m i^m J_m(kr)\, e^{im(\varphi-\varphi_k)}$$

$$\lim_{r\to 0} \int \frac{d\varphi'}{\sqrt{2\pi}} e^{ikr\cos\varphi'} e^{im\varphi'} = \left(\frac{ikr}{2}\right)^m \frac{1}{m!}$$

$$\downarrow$$

$$\boxed{C_m(k) = \frac{(-i)^m}{k} \frac{1}{\sqrt{2\pi}}}$$

3d

$$|k,\ell,m\rangle \longrightarrow \psi_{k\ell m}(\vec r) = j_\ell(kr)\, Y_{\ell m}(\theta,\varphi)$$

$$|\vec k\rangle \longrightarrow \phi_{\vec k}(\vec r) = \frac{1}{(2\pi)^{3/2}} e^{i\vec k\vec r}$$

$$\langle \vec k | k', \ell m\rangle \propto \delta(k-k')\, Y_{\ell m}(\theta_{\vec k}, \varphi_{\vec k})$$

$$e^{i\vec k\vec r} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} C_\ell(k)\, Y_{\ell m}(\theta_k, \varphi_k)\, j_\ell(kr)\, Y_{\ell m}(\theta,\varphi)$$

$$C_\ell(k)\frac{2\ell+1}{2} j_\ell(kr) = \frac{2\ell+1}{2} \int_{-1}^{1} dx\, e^{ikrx} P_\ell(x)$$

$$r\to 0;\ j_\ell(kr) \to \frac{(kr)^\ell}{(2\ell+1)!!}$$

$$\lim_{r\to 0} \int dx\, e^{ikrx} P_\ell(x) = \frac{(ikr)^\ell}{(2\ell+1)!!}\, 2$$

$$\boxed{C_\ell(k) = 4\pi i^\ell} \qquad e^{i\vec k\vec r} = 4\pi \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} i^\ell\, Y^*_{\ell m}(\theta_{\vec k}, \varphi_{\vec k})\, j_\ell(kr)\, Y_{\ell m}(\theta,\varphi)$$

m dado

$$\downarrow$$

$$= 4\pi \sum_{\ell=0}^{\infty} i^\ell (2\ell+1)\, P_\ell(\cos\gamma_{\vec r\vec k})\, j_\ell(kr)$$

$$\int_0^{2\pi} mx\cos\varphi\, e^{ix\operatorname{Sen}\varphi - im\varphi}\, d\varphi =$$

$$\cancel{u = \cos e^{ix\operatorname{Sen}\varphi - im}} \; ; \; \cancel{dv = \cos\varphi / mx}$$

$$\cancel{du = i(x\operatorname{Sen}\varphi - m)e^{ix\operatorname{Sen}\varphi - im}} \quad \cancel{v = \operatorname{Sen}\varphi / mx}$$

$$\cancel{\operatorname{Sen}\varphi\, mx\, e^{ix\operatorname{Sen}\varphi - im}\Big|_0^{2\pi} + \int \operatorname{Sen}\varphi\, mx\, i(x\operatorname{Sen}\varphi - m)e^{ix\operatorname{Sen}\varphi - im} =}$$

$$\cancel{= i\int (x^2\operatorname{Sen}^2\varphi\, m - m^2 x\operatorname{Sen}\varphi)e^{ix\operatorname{Sen}\varphi - im}\, d\varphi =}$$

$$u = e^{-im\varphi} \; ; \; dv = \cos\varphi\, e^{ix\operatorname{Sen}\varphi}$$

$$du = -im e^{-im\varphi} \quad v = \frac{e^{ix\operatorname{Sen}\varphi}}{ix}$$

$$= \frac{\cancel{e^{-im\varphi}}\, e^{ix\operatorname{Sen}\varphi}}{ix}\Bigg|_0^{2\pi} + \int_0^{2\pi}\left[\frac{e^{ix\operatorname{Sen}\varphi - im\varphi}}{\cancel{x}}\, m\, d\varphi\right] m\cancel{x} =$$

$$e^{-im2\pi} = 1$$

$$= \int_0^{2\pi} m^2 (e^{ix\operatorname{Sen}\varphi - im\varphi})\, d\varphi$$

4

$$\underbrace{e^{ikr\cos\varphi}} = \sum_m i^m J_m(kr) e^{im\varphi}$$

$$\cos\varphi - \pi/2 = \operatorname{Sen}\varphi + \pi/2 \qquad \varphi \pm \pi/2 = \theta$$

$$e^{ix\operatorname{Sen}\theta} = \sum_m i^m J_m(kr)e^{im\theta} = \sum_m J_m(kr)e^{im\varphi + \frac{im\pi}{2}} =$$

$$= \boxed{\sum_m J_m(kr)e^{im\varphi}\, i^m} \qquad i^m = e^{im\pi/2}$$

$$\int_0^{2\pi} mx \cos\varphi \, e^{ix \operatorname{Sen}\varphi - im\varphi} \, d\varphi =$$

$$\cancel{u = \cos\varphi \, e^{ix\operatorname{Sen}\varphi - im}} \; ; \; \cancel{dv = \cos\varphi / mx}$$

$$\cancel{du = i(x\operatorname{Sen}\varphi - m) e^{ix\operatorname{Sen}\varphi - im}} \qquad \cancel{v = \operatorname{Sen}\varphi / mx}$$

$$\cancel{\operatorname{Sen}\varphi \, mx \, e^{ix\operatorname{Sen}\varphi - im}\Big|_0^{2\pi} + \int \operatorname{Sen}\varphi \, mx \, i(x\operatorname{Sen}\varphi - m) e^{ix\operatorname{Sen}\varphi - im}} =$$

$$\cancel{= i\int (x^2 \operatorname{Sen}^2\varphi \, m - m^2 x \operatorname{Sen}\varphi) e^{ix\operatorname{Sen}\varphi - im} \, d\varphi} =$$

$$u = e^{-im\varphi} \; ; \; dv = \cos\varphi \, e^{ix\operatorname{Sen}\varphi}$$

$$du = -im \, e^{-im\varphi} \qquad v = \frac{e^{ix\operatorname{Sen}\varphi}}{ix}$$

$$= \frac{\cancel{e^{-im\varphi}} e^{ix\operatorname{Sen}\varphi}}{ix}\Bigg|_0^{2\pi} + \int_0^{2\pi} \left[\frac{e^{ix\operatorname{Sen}\varphi} \, e^{-im\varphi}}{\cancel{x}} m \, d\varphi \right] m\cancel{x} =$$

$$e^{-im2\pi} = 1$$

$$= \int_0^{2\pi} m^2 (e^{ix\operatorname{Sen}\varphi - im\varphi}) \, d\varphi$$

(4)

$$\underbrace{e^{ikr\cos\varphi}} = \sum_m i^m J_m(kr) e^{im\varphi}$$

$$\cos \varphi - \pi/2 = \operatorname{Sen}\varphi + \pi/2 \qquad \varphi \pm \pi/2 = \theta$$

$$e^{ix\operatorname{Sen}\theta} = \sum_m J_m(kr) e^{im\theta} = \sum_m J_m(kr) e^{im\varphi + \frac{im\pi}{2}} =$$

$$= \boxed{\sum_m J_m(kr) e^{im\varphi} i^m} \qquad i^m = e^{im\pi/2}$$

③

$$g(x) = \frac{1}{2\pi}\int_0^{2\pi} e^{-im\varphi} e^{ix\,Sen\,\varphi}$$

$$x^2 y'' + xy' + (x^2 - m^2)y = 0$$

$$g'(x) = \frac{1}{2\pi}\int_0^{2\pi} e^{-im\varphi} e^{ix\,Sen\,\varphi}\, Sen\,\varphi$$

$$g''(x) = -\frac{1}{2\pi}\int_0^{2\pi} e^{-im\varphi} e^{ix\,Sen\,\varphi}\, Sen^2\varphi$$

$$-g'(x) = \frac{1}{2\pi}\int_0^{2\pi} i e^{-im\varphi} e^{ix\,Sen\,\varphi}\, Sen\,\varphi$$

$$u = e^{i(x\,Sen\,\varphi - im)} \quad ; \quad dv = Sen\,\varphi$$

$$du = -1(x\,Cos\,\varphi - im)\, e^{i(x\,Sen\,\varphi - im)} \quad ; \quad v = -Cos\,\varphi$$

$$g'(x) = \frac{1}{2\pi}\left[e^{i(x\,Sen\,\varphi - im)} (-Cos\,\varphi) \right]_0^{2\pi} + \int_0^{2\pi} i\,Cos\,\varphi\,(x\,Cos\,\varphi - im)\, e^{i(x\,Sen\,\varphi - im)}$$

$$g'(x) = \int_0^{2\pi} -i\,Cos\,\varphi\,(x\,Cos\,\varphi - im)\, e^{i(x\,Sen\,\varphi - im)} \quad \frac{1}{2\pi}$$

$$-\frac{x^2}{2\pi}\int e^{-im\varphi} \cancel{e^{ix\,Sen\,\varphi}}\, Sen^2\varphi + \int (\cancel{1} - \cancel{Sen^2\varphi})x^2 - mx\,Cos\,\varphi)\, e^{-im\varphi + ix\,Sen\,\varphi}\, d\varphi$$

$$+\frac{x^2}{2\pi}\int_0 \cancel{e^{-im\varphi}}\, e^{ix\,Sen\,\varphi} - \frac{m^2}{2\pi}\int e^{-im\varphi} e^{ix\,Sen\,\varphi}\, d\varphi = 0$$

$$\frac{1}{2\pi}\int_0^{2\pi} mx\,Cos\,\varphi\, e^{im\varphi + ix\,Sen\,\varphi}\, d\varphi = \frac{m^2}{2\pi}\int_0^{2\pi} e^{-im\varphi} e^{ix\,Sen\,\varphi}\, d\varphi$$

③

$$g(x) = \frac{1}{2\pi}\int_0^{2\pi} e^{-im\varphi} e^{ix\,\mathrm{Sen}\,\varphi}$$

$$x^2y'' + xy' + (x^2 - m^2)y = 0$$

$$g'(x) = \frac{1}{2\pi}\int_0^{2\pi} e^{-im\varphi} e^{ix\,\mathrm{Sen}\,\varphi}\,\mathrm{Sen}\,\varphi$$

$$g''(x) = -\frac{1}{2\pi}\int_0^{2\pi} e^{-im\varphi} e^{ix\,\mathrm{Sen}\,\varphi}\,\mathrm{Sen}^2\varphi$$

$$-g'(x) = \frac{1}{2\pi}\int_0^{2\pi} i e^{-im\varphi} e^{ix\,\mathrm{Sen}\,\varphi}\,\mathrm{Sen}\,\varphi$$

$$u = e^{i(x\,\mathrm{Sen}\,\varphi - \cancel{i}m)} \quad ; \quad dv = \mathrm{Sen}\,\varphi$$

$$du = i(x\cos\varphi - im)\,e^{i(x\,\mathrm{Sen}\,\varphi - im)} \quad ; \quad v = -\cos\varphi$$

$$g'(x) = \frac{1}{2\pi}\left[e^{i(x\,\mathrm{Sen}\,\varphi - im)}(\cancel{-\cos\varphi})\right]_0^{2\pi} + \int_0^{2\pi} i\cos\varphi\,(x\cos\varphi - im)\,e^{i(x\,\mathrm{Sen}\,\varphi - im)}$$

$$g'(x) = \int_0^{2\pi} -i\cos\varphi\,(x\cos\varphi - im)\,e^{i(x\,\mathrm{Sen}\,\varphi - im)} \quad \frac{1}{2\pi}$$

$$-\frac{x^2}{2\pi}\int e^{-im\varphi}\,\cancel{e^{ix\,\mathrm{Sen}\,\varphi}}\,\mathrm{Sen}^2\varphi - \int \cos^2\varphi\,(\cancel{1 - \mathrm{Sen}^2\varphi})\,x^2 - mx\cos\varphi)\,e^{-im\varphi + ix\,\mathrm{Sen}\,\varphi}\,d\varphi$$

$$+\frac{x^2}{2\pi}\int_0 \cancel{e^{-im\varphi}}\,e^{ix\,\mathrm{Sen}\,\varphi} - \frac{m^2}{2\pi}\int e^{-im\varphi} e^{ix\,\mathrm{Sen}\,\varphi}\,d\varphi = 0$$

$$\frac{1}{2\pi}\int_0^{2\pi} mx\cos\varphi\, e^{im\varphi + ix\,\mathrm{Sen}\,\varphi}\,d\varphi = \frac{m^2}{2\pi}\int_0^{2\pi} e^{-im\varphi} e^{ix\,\mathrm{Sen}\,\varphi}\,d\varphi$$

$$\int_0^a [J_m(x)]^2 x\,dx = \frac{1}{2}a^2[J_m'(a)]^2 + \frac{1}{2}(a^2-m^2)[J_m(a)]^2$$

usando la demostración anterior

$$\int_0^a [J_m(x)]^2 x\,dx = \int_0^a \left[(x^2 J_m'(x))^2 + (x^2-m^2)J_m^2(x)\right]' dx\,\frac{1}{2} =$$

$$= \left[(x^2 J_m'(x))^2 + (x^2-m^2)J_m^2(x)\right]\Big|_0^a = \frac{1}{2}\left(a^2 J_m'^2(a) + (a^2-m^2)J_m^2(a)\right)$$

$J_m(0) = J_m'(0) = 0$

Dirichlet

$$b_{nm} = \frac{1}{2}a^2[J_{m+1}(x_{nm})]^2 \qquad (J_m(x_{nm}) = 0)$$

$$b_{nm} = \int_0^a r\,dr\,[J_m(k_{nm}r)]^2$$

$k\,a = x$

$J_m(x) = 0$

$dr = k\,dx$

$$b_{nm} = \int_0^a \frac{x\,dx}{k_{nm}^2}[J_m(x_{nm})]^2 = \frac{1}{2}a^2[J_m'(x)]^2 + \frac{1}{2}(a^2-m^2)[\cancel{J_m(0)}]^2$$

$$J_m'(x_{nm})^2 = -J_{m+1} + \frac{m}{x}\cancel{J_m(x)} = [J_{m+1}(x_{nm})]^2$$

Neumann

$$b_{nm} = \frac{1}{2}a^2\left(1-\frac{m^2}{x_{nm}^2}\right)[J_m(x_{nm})]^2 \qquad J_m'(x_{nm}) = 0$$

$$\int_0^a \frac{x_{nm}\,dt}{k_{nm}^2} = \frac{1}{2}a^2[\cancel{J_m'(x_{nm})}]^2 + \frac{1}{2k_{nm}^2}(x_{nm}^2-m^2)[J_m(x_{nm})]^2 =$$

$$= \boxed{\frac{a^2}{2}\left(1-\frac{m^2}{x_{nm}^2}\right)[J_m(x_{nm})]^2}$$

$$\int_0^a [J_m(x)]^2 x\,dx = \frac{1}{2}a^2[J_m'(a)]^2 + \frac{1}{2}(a^2-m^2)[J_m(a)]^2$$

usando la demostración anterior

$$\int_0^a [J_m(x)]^2 x\,dx = \int_0^a [(x^2 J_m'(x))^2 + (x^2-m^2) J_m^2(x)]' dx \frac{1}{2} =$$

$$= [(x^2 J_m'(x))^2 + (x^2-m^2) J_m^2(x)]\Big|_0^a = \frac{1}{2}(a^2 J_m'^2(a) + (a^2-m^2) J_m^2(a))$$

$J_m(0) = J_m'(0) = 0$

Dirichlet

$$b_{nm} = \frac{1}{2}a^2[J_{m+1}(x_{nm})]^2 \quad (J_m(x_{nm})) = 0$$

$$b_{nm} = \int_0^a r\,dr [J_m(K_{nm} r)]^2 \qquad k^a a^a = x$$

$J_m(x) = 0$

$dr = K\,dx$

$$b_{nm} = \int_0^a \frac{x\,dx}{K_{nm}^2}[J_m(x_{nm})]^2 = \frac{1}{2}a^2[J_m'(x)]^2 + \frac{1}{2}(a^2-m^2)[\cancel{J_m}(0)]^2$$

$$J_m'(x_{nm})^2 = -J_{m+1} + \frac{m}{x}\cancel{J_m}(x) = [J_{m+1}(x_{nm})]$$

Neumann

$$b_{nm} = \frac{1}{2}a^2\left(1-\frac{m^2}{x_{nm}^2}\right)[J_m(x_{nm})]^2 \quad J_m'(x_{nm}) = 0$$

$$\int_0^a \frac{x_{nm}\,dt}{K_{nm}^2} = \frac{1}{2}a^2[\cancel{J_m'}(x_{nm})]^2 + \frac{1}{2K_{nm}^2}(x_{nm}^2-m^2)[J_m(x_{nm})]^2 =$$

$$= \boxed{\frac{a^2}{2}\left(1-\frac{m^2}{x_{nm}^2}\right)[J_m(x_{nm})]^2}$$

2

$$\left[(xy')^2 + (x^2 - m^2)y^2\right]' = 2xy^2$$

$$-[xy'] + \frac{m^2}{x}\, y' = xy \quad \text{Ec. Bessel SL}$$

$$-(xy')^2 + \frac{m^2}{x} x y y' = (xy')^2 \quad y = \frac{2x}{m^2}$$

$$m^2 y y' 2 = 2(xy')^2$$

$$-[xy'][xy'] + m^2 y(y') = y y' x^2$$

$$\int (xy')' \to \quad \begin{matrix} xy' = u \\ d[xy'] = du \end{matrix} \quad \int u\,du = \frac{u^2}{2} = \frac{(xy')^2}{2}$$

$$\int m^2 y(y') \to \quad \begin{matrix} y = u \\ y' = du \end{matrix} \quad \int u\,du = \frac{u^2}{2} = \frac{(y)^2}{2} m^2$$

$$\int y y' x^2 \quad \begin{matrix} u = x^2 & du = 2x \\ dv = yy' & v = \frac{y^2}{2} \end{matrix} \quad \to \quad \frac{x^2 y^2}{2} - \int y^2 x$$

$$\frac{1}{2}\left[(xy')^2 + (x^2 - m^2)\, y^2\right] = \cancel{2xy^2} = \int y^2 x$$

derivando:

$$\boxed{\left[(xy')^2 + (x^2 - m^2)\, y^2\right]' = 2y^2 x}$$

(2)

$$\left[(xy')^2 + (x^2 - m^2)y^2 \right]' = 2xy^2$$

$$-[xy'] + \frac{m^2}{x}\,y' = xy \qquad \text{Ec. Bessel SL}$$

$$\cancel{-(xy')^2 + \frac{m^2}{x}\,x\,y\,y' = (xy')^2 \qquad y = \frac{2x}{m^2}}$$

$$\cancel{m^2\, y\, y' = 2(xy')^2}$$

$$-[xy'][xy'] + m^2 y(y') = yy'x^2$$

$$\int (xy')' \rightarrow \begin{matrix} xy' = u \\ d[xy'] = du \end{matrix} \qquad \int u\,du = \frac{u^2}{2} = \frac{(xy')^2}{2}$$

$$\int m^2\, y(y') \rightarrow \begin{matrix} y = u \\ y' = du \end{matrix} \qquad \int u\,du = \frac{u^2}{2} = \frac{(y)^2}{2}\,m^2$$

$$\int yy'x^2 \qquad \begin{matrix} u = x^2 & du = 2x \\ dv = yy' & v = \frac{y^2}{2} \end{matrix} \rightarrow \frac{x^2y^2}{2} - \int y^2 x$$

$$\frac{1}{2}\left[(xy')^2 + (x^2 - m^2)\, y^2 \right] = \cancel{2xy^2} = \int y^2 x$$

derivando:

$$\boxed{\left[(xy')^2 + (x^2 - m^2)\, y^2 \right]' = 2y^2x}$$

$$= \int_0 m^2 \left(e^{ix\,Sen\varphi - im\varphi}\right) d\varphi$$

4

$$\underbrace{e^{ikr\,Cos\varphi}}_{} = \sum_m i^m J_m(kr)\, e^{im\varphi}$$

$$Cos\,\varphi \cancel{+\pi/2} = Sen\,\varphi + \pi/2 \qquad \varphi \pm \pi/2 = \theta$$

$$e^{ix\,Sen\,\theta} = \sum_m \cancel{i^m}\, J_m(kr)\, e^{im\theta} = \sum_m J_m(kr)\, e^{im\varphi + \frac{im\pi}{2}} =$$

$$= \boxed{\sum_m J_m(kr)\, e^{im\varphi}\, i^m} \qquad i^m = e^{im\pi/2}$$

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